

Global Anomalies in the Batalin Vilkovisky Quantization

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Abstract

The Batalin Vilkovisky (BV) quantization provides a general procedure for calculating anomalies associated to gauge symmetries. Recent results show that even higher loop order contributions can be calculated by introducing an appropriate regularization-renormalization scheme. However, in its standard form, the BV quantization is not sensible to quantum violations of the classical conservation of Noether currents, the so called global anomalies. We show here that the BV field antifield method can be extended in such a way that the Ward identities involving divergencies of global Abelian currents can be calculated from the generating functional, a result that would not be obtained by just associating constant ghosts to global symmetries. This extension, consisting of trivially gauging the global Abelian symmetries, poses no extra obstruction to the solution of the master equation, as it happens in the case of gauge anomalies. We illustrate the procedure with the axial model and also calculating the Adler Bell Jackiw anomaly.

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1 Introduction

The Batalin Vilkovisky (BV) (or Field-Antifield) method is a Lagrangian path integral quantization scheme for general gauge theories[1, 2]. At classical level (zero order in \hbar), given a field theory and the associated gauge algebra, one has a systematic approach of building up a gauge fixing structure, even when it involves a chain of ghosts for ghosts, as in the case of reducible theories. This procedure generalizes the original idea of Faddeev and Popov[3]. The condition of BRST[4, 5] invariance, at this level, is translated in the so called classical master equation. This equation is mathematically well defined and needs no regularization procedure. Several important results related to this classical level of the BV formalism are reviewed in the recent literature[6, 7, 8].

At the quantum level the situation is different. The quantum master equation is, in principle, just formal, as it involves the ill-defined Δ operator associated to the behavior of the path integral measure. The Pauli Villars(PV) regularization procedure was successfully applied to BV in[9], in such a way that one arrives at a well defined interpretation to the one loop order equation. This important step was the starting point for a series of results related to calculating gauge anomalies and Wess Zumino terms at this one loop level, that are reviewed, for example, in [8].

The question of corrections of loop order higher than one in the BV quantization, where the PV regularization can not be applied, has been object of very recent investigations. One proposal for making sense of higher loop BV is the use of the non local regularization, in such a way that the action of the operator Δ is not singular[10]. A different approach is to translate the master equation in relations that do not involve the operator Δ and then use the BPHZ renormalization scheme[11]. Both approaches allow the calculation of gauge anomalies at higher loops.

In contrast to this large improvement in people's ability to calculate gauge anomalies using BV quantization, global anomalies are simply ignored if one follows the standard approach. One normally associates ghost fields with the parameters of local symmetries. Global symmetries may be described by the introduction of constant ghosts. This kind of approach, with a wide list of important related references can be found, for example, in [12, 13]. In the specific case of the Field Antifield quantization, the introduction of constant ghosts in order to derive anomaly free Ward identities for theories with both gauge and global symmetries forming a general algebra was discussed in [14, 15].

If one tries to calculate global anomalies³ in the field antifield formalism with the aid of constant ghosts one finds a vanishing result. The point is that anomalies appear in the BV formalism multiplied by the corresponding ghosts and integrated over space-time. As global anomalies can be total derivatives [16], as for example the axial anomaly, proportional to $\epsilon^{\mu\nu\rho\sigma} Tr(F_{\mu\nu}F_{\rho\sigma})$, their contributions to ΔS could vanish upon space time integrations, or possibly give a constant value, that can be absorbed in the normalization, as in the case of topologically non trivial solutions.

³We are calling as "global anomalies" the quantum violation in the conservation of Noether currents, and NOT the breaking of global symmetries, that we assume not to happen

Anyway the constant ghosts would not lead to new field dependent terms in ΔS , that could reflect the possible non trivial quantum behavior of the theory regarding the global symmetry. Therefore, associating constant ghosts to this symmetries would give no extra contribution to the master equation. As a consequence, the BV generating functional would not generate the appropriate anomalous Ward identities involving the divergencies of the Noether currents. Therefore, celebrated results, as the Adler Bell Jackiw[17, 18] anomaly of (non chiral) fermions coupled to gauge fields cannot be calculated in the standard BV framework by just introducing constant ghosts. Although the path integral computation of global anomalies was achieved almost two decades ago in refs. [19], by calculating the regularized Jacobian of the associated local transformations, the incorporation of such a procedure in the BV context is clearly still lacking.

Considering some classical action with a global Abelian symmetry with closed and irreducible algebra, we will enlarge minimally the theory field space in such a way that a new action will be found, with a larger gauge symmetry. In this extended action the original global Abelian symmetry is realized locally, but the original theory is recovered at some gauge. The field-antifield formalism can then be applied in the usual way, but now the one loop order master equation will get a non vanishing extra contribution associated to the original global anomaly. We will see that this contribution will be canceled by an appropriate counterterm, in contrast to what happens with essential gauge anomalies, where one can not find counterterms that solve the master equation. However, the counterterms associated with global transformations will have a non trivial role. They will contribute to the Ward identities with insertions of divergencies of the classically conserved global currents, giving the appropriate one loop anomalous corrections.

The article is organized in the following way: in section (2) we briefly review some results of the BV quantization related to anomalous theories. In section (3) we present our general approach to calculate global Abelian anomalies in BV, illustrating it with the axial model in section (4). The axial anomaly of non chiral fermions coupled to gauge fields, the so called Adler, Bell, Jackiw anomaly, is calculated in section (5). Some concluding remarks are left to section (6).

2 Anomalies in the standard BV quantization

There are presently in the literature a reasonable amount of reviews about BV quantization and (or) its application to anomalous gauge theories, as [6, 7, 8, 20, 21]. We will thus present just a brief summary of results to be used in the following sections, stressing some specific points, like gauge fixing. The quantum action $W[\phi^A, \phi_A^*]$ is defined in an enlarged space where the set ϕ^A includes the classical fields ϕ^i plus all the fields possibly required for gauge fixing: ghosts c^α , antighosts, auxiliary fields, ghosts for ghosts, ... , and ϕ_A^* are the corresponding antifields, each one with Grassman parity opposite to the corresponding field. The generating functional is built up as

$$Z_\Psi[J] = \int \prod D\phi^A \exp \frac{i}{\hbar} \left(W[\phi^A, \phi_A^* = \frac{\partial \Psi}{\partial \phi^A}] + J_A \phi^A \right) \quad (1)$$

and the expectation value for an operator X is calculated as:

$$\langle X \rangle_{\Psi, J} = \int \prod D\phi^A X \exp \left(\frac{i}{\hbar} W[\phi^A, \phi_A^* = \frac{\partial \Psi}{\partial \phi^A}] + J_A \phi^A \right) \quad (2)$$

The condition that (1) does not depend on the gauge choice (represented by the gauge fixing fermion Ψ) when the sources are not present, or when the source term is BRST invariant, is translated in the so called master equation:

$$\langle \frac{1}{2}(W, W) - i\hbar \Delta W \rangle_{\Psi, J} = 0, \quad (3)$$

where we are explicitly calling the attention to the fact that this equation comes in as an expectation value. In Eq. (3) the antibracket is defined as $(X, Y) = \frac{\delta_r X}{\delta \phi^A} \frac{\delta_l Y}{\delta \phi_A^*} - \frac{\delta_r X}{\delta \phi_A^*} \frac{\delta_l Y}{\delta \phi^A}$ and the operator Delta as $\Delta \equiv \frac{\delta_r}{\delta \phi^A} \frac{\delta_l}{\delta \phi_A^*}$. We are using the de Witt notation of sum and integration over space-time variables for repeated indices, when pertinent. It is also useful to observe that in fact Eq. (3) is deduced as an expectation value like (2) and so it is implicitly assumed that there exist some Ψ that fixes properly the redundant degrees of freedom associated to gauge invariance. This point will be important in the developments we are going to present in the next sections.

The operator Δ involves a double functional derivative in the same space-time point. Therefore, acting on local functionals, it leads to a singular $\delta(0)$. That is why, as we said in the introduction, the master equation at loop order equal or greater than one is just formal and a regularization scheme must be introduced. Expanding the quantum action in a power series in \hbar : $W[\phi^A, \phi_A^*] = S[\phi^A, \phi_A^*] + \sum_{p=1}^{\infty} \hbar^p M_p[\phi^A, \phi_A^*]$ we can write also the master equation (3) in loop order. The two first terms are:

$$(S, S) = 0 \quad (4)$$

$$(M_1, S) = i \Delta S \quad (5)$$

The zero loop order action S must satisfy the boundary condition: $S[\phi^A, \phi_A^* = 0] = S_0[\phi^i]$ where $S_0(\phi^i)$ is the original classical action. Considering an irreducible gauge theory where the original local symmetries are of the form:

$$\delta \phi^i = R_\alpha^i(\phi) \theta^\alpha \quad (6)$$

with θ^α space time dependent parameters, gauge fixing is obtained by enlarging the field content of the theory, associating ghosts c^α to the local parameters, and requiring that

$$\frac{\delta_r \delta_l S[\phi, \phi^*]}{\delta c^\alpha \delta \phi_i^*} \Big|_{\phi^*=0} = R_\alpha^i(\phi). \quad (7)$$

Eq. (4) contains all the classical gauge structure of the original theory. Quantum obstructions of the gauge invariance may appear at one loop order and beyond. As it was already observed, they need to be regularized. The one loop order equation may be regularized using the Pauli Villars procedure [9, 21, 7, 8]. When there is no local M_1 term in the original space of fields that solves eq. (5), the theory has a gauge (or local) anomaly. The violation of (5), for the case of irreducible theories with closed gauge algebra, may be written as

$$\mathcal{A}[\phi, \phi^*] = \Delta S + \frac{i}{\hbar}(S, M_1) = a_\alpha c^\alpha \quad (8)$$

and translates the possible non trivial behavior of the path integral measure with respect to some of the local symmetries. Actually the form of (8) depends both on the regularization procedure used to calculate ΔS and on the counterterms M_1 that one is choosing.

3 Global Abelian Anomalies in the BV Quantization

Let us consider a classical action $S_0[\phi^i]$, invariant under local transformations $\delta\phi^i = R_\alpha^i[\phi]\theta^\alpha(x)$ and therefore satisfying the Noether identities

$$\frac{\delta S_0}{\delta \phi^i} R_\alpha^i = 0. \quad (9)$$

Besides the local invariances, that we assume to be non anomalous, the action S_0 has possibly a large number of global symmetries also [14, 15]. We will not be concerned with the whole set of global transformations that leave S_0 invariant, but just investigate a particular Abelian subset associated to global anomalies and satisfying some properties that, as we will see, will hold for very important cases, like the axial Abelian anomaly. Let us split the set of classical fields ϕ^i into two subsets $\phi^i = \{A^m, \psi^r\}$, such that the fields A^m are invariant under the global transformations considered. Thus, writing together the global and local infinitesimal transformations we have

$$\delta A^m = R_\alpha^m[\phi] \theta^\alpha(x), \quad (10)$$

$$\delta \psi^r = R_\alpha^r[\phi] \theta^\alpha(x) + \overline{R}_a^r[\phi] \epsilon^a, \quad (11)$$

where ϵ^a are constant parameters.

We will now assume that the fields ψ transform exponentially. In other words, the finite version of relation (11) has the specific form

$$\psi'^r[\psi, \theta, \epsilon] = \exp\{i(T^\alpha \theta^\alpha(x) + \bar{T}^a \epsilon^a)\}_s^r \psi^s \quad (12)$$

where it should be noted that we are using the same notation for the parameters θ and ϵ as in the infinitesimal case (11) just to simplify the notation.

Transformations like (12) are what occurs, for instance, in QCD if one is concerned only with the phase transformations of the fermionic fields, and not with the Poincaré transformations (where global anomalies are absent). In that case the gauge fields transform only locally while the matter fields transform locally and globally under the gauge as well as the global axial symmetries of the action.

We will furthermore assume that the global transformations are Abelian. That means:

$$[\bar{T}^a, \bar{T}^b] = 0 = [T^\alpha, \bar{T}^a] \quad (13)$$

The infinitesimal expression (11) can, of course, be obtained from (12) by Taylor expansions in θ^α and ϵ^a . Thus we identify the generators as

$$\begin{aligned} R_\alpha^r &= \frac{\partial \psi'^r}{\partial \theta^\alpha} \Big|_{\theta=\epsilon=0} = i [T^\alpha \psi]^r, \\ \bar{R}_a^r &= \frac{\partial \psi'^r}{\partial \epsilon^a} \Big|_{\theta=\epsilon=0} = i [\bar{T}^a \psi]^r. \end{aligned} \quad (14)$$

At classical level, a gauged version for the global subset of the symmetries appearing in (11) can be written. More precisely, we can redefine the theory in such a way that the set of parameters ϵ^a becomes space-time dependent and, at the same time, it is imposed that the old theory is recovered in some gauge. In order to reach this goal we introduce collective fields χ^a [25] and define (see (12))

$$\bar{\psi}^r \equiv \psi'^r[\psi, 0, \chi] = \left[\exp\{i \bar{T}^a \chi^a\} \right]_s^r \psi^s \quad (15)$$

By using (11)-(15), we see that $\bar{\psi}^r$ is invariant under the transformation associated to the parameter $\epsilon^a(x)$, now made local, if we impose that the new fields χ also transform appropriately. The new gauge transformations that leave (15) invariant read:

$$\begin{aligned} \delta \psi^r &= i [\bar{T}^a \epsilon^a(x) \psi]^r \\ \delta \chi^a &= -\epsilon^a(x). \end{aligned} \quad (16)$$

At this point we extend the action $S_0[\phi^i] = S_0[A^m, \psi^r]$ to a new action $S_1[A^m, \psi^r, \chi^a]$ in such a way that

$$S_1[A, \psi, \chi] \equiv S_0[A, \bar{\psi}]. \quad (17)$$

As S_1 depends on ψ and χ only through $\bar{\psi}$, it becomes clear that the action S_1 is invariant under the transformations parameterized by $\epsilon^a(x)$. Therefore

$$\delta S_1[\phi, \chi]_{|\theta=0} = 0. \quad (18)$$

We observe that this is true only because we are assuming that the original global symmetries are Abelian (eqs. (13)).

Now, the process of gauging the symmetry associated to the set of parameters ϵ^a does not modify the form of the generators R_α^m and R_α^r . Then

$$\delta S_1[\phi, \chi]_{|\epsilon=0} = \frac{\delta S_0[\bar{\psi}, A]}{\delta \bar{\psi}^r} \delta \bar{\psi}^r + \frac{\delta S_0[\bar{\psi}, A]}{\delta A^m} \delta A^m, \quad (19)$$

and therefore the Noether identities (9) of the original local gauge sector continue to be valid. This means that it is possible to gauge rigid symmetries of $S_0[\phi^i]$ of the kind expressed by (12) without destroying its original local ones, as long as (13) holds also.

An important point to be remarked is that $S_1[A, \psi, \chi]$ recovers $S_0[\phi]$ in the gauge $\chi^a = 0$. This assures that the original theory, at least at the classical level, is not changed. So, we have succeed in gauging some global symmetries of a classical action S_0 , submitted to the quoted restrictions, by enlarging the configuration space in such a way that the original theory is recovered when the new fields are set to zero. Although this is a simple construction, the existence of the new local symmetries of the classical action S_1 will enable a non trivial incorporation of global anomalies in the field antifield formalism. As we are going to show, this gives us room for introducing *non-constant* ghosts associated to global symmetries. This will be essential in the process of building up anomalous Schwinger-Dyson equations reflecting the non conservation of Noether currents.

As we are assuming that S_0 is invariant under the global symmetry appearing in (11), we have[16]:

$$\frac{\partial \mathcal{L}_1}{\partial \chi^a} = 0, \quad (20)$$

where \mathcal{L}_1 is the Lagrangian density associated to S_1 . This implies that

$$\frac{\delta S_1}{\delta \chi^a} \Big|_{\chi^a=0} = -\partial_\mu J_a^\mu \quad (21)$$

where the current

$$J_a^\mu = \frac{\partial \mathcal{L}_1}{\partial (\partial_\mu \chi^a)} \Big|_{\chi^a=0} \quad (22)$$

is just the on shell classically conserved Noether current. In the derivation of (21) we are assuming that S_0 does not depend on higher derivatives in ϕ . Result (21) will be important in the calculation of the anomalous non conservation of currents (22).

At this point it is interesting to compare the introduction of the fields χ^a here with the introduction of fields associated to gauge group elements in order to generate Wess Zumino(WZ) terms for anomalous gauge theories as it was done first

for chiral QCD2 in [22] and then generalized in [23]. In these articles, where true gauge anomalies are present, one considers extra WZ fields that are not present at the classical level but will show up just at the one loop order term of the action (M_1). It was shown in [24] that if one appropriately includes ghost fields associated to the invariance of the classical action with respect to these extra WZ fields one would get the result that gauge anomalies are never canceled by the Wess Zumino terms but just shifted to other symmetry.

Here we have a completely different situation. Our classical action S_1 is NOT independent of the extra fields χ^a . Although (20) may seem to indicate it, we realize by (22) that S_1 depends on χ through its space time derivatives. So, we can not transform χ arbitrarily as it happens with the WZ fields in [22, 23, 24]. Actually, we can see in (16) that the new gauge invariance corresponds to a simultaneous change in ψ and χ . We will gauge fix it with extra ghosts, but we do not have a shift symmetry in χ like those discussed in [24]. It is important to note that here, in contrast to [22, 23, 24], we are considering theories with no gauge anomalies. Thus, our procedure for detecting the anomalous violation of Noether currents should not make our theory become gauge anomalous. We are just building up an enlarged gauge theory that reproduces the original (non anomalous) one for some partial gauge fixing. If, as assumed, the master equation was solvable in the original theory, the same thing is expected to hold in the enlarged theory. The important difference is that, as we will see, we will have to add a counterterm that will generate the Green functions with the insertion of the divergence of the local current considered.

In order to quantize the theory along the field-antifield line, we introduce, besides the usual ghosts c^α corresponding to the original gauge symmetries of $S_0[\phi^i]$, new non constant Abelian ghosts C^a corresponding to the symmetry (16) of $S_1[\phi^i, \chi^a]$, as well as their corresponding antifields. The classical BV action has then the general form:

$$\begin{aligned} S[\varphi^I, \varphi_I^*] &= S_1[\phi^i, \chi^a] + \phi_i^* R_\alpha^i c^\alpha - c_\alpha^* T_{\beta\gamma}^\alpha c^\gamma c^\beta \\ &+ \psi_r^* \bar{R}_a^r C^a - \chi_a^* C^a + \bar{\pi}_a \bar{C}^{*a} + \dots \end{aligned} \quad (23)$$

where we have introduced the notation $\{\varphi^I\} \equiv \{\phi^A, \chi^a, C^a, \bar{\pi}^a, \bar{C}^a\}$ for the complete set of fields. In the above equation the T 's are structure constants associated to the original gauge algebra, which we are assuming that is closed, irreducible and disjoint. It is worth to mention that as in eq. (16) the parameter is local, the ghosts C (as well as c) are also local, in contrast to the standard approach of associating constant ghosts to global symmetries.

Anomalies then formally come from

$$\triangle S = \frac{\delta R_\alpha^i}{\delta \phi^i} c^\alpha + \frac{\delta \bar{R}_a^i}{\delta \phi^i} C^a \quad (24)$$

but this expression is actually ill-defined, as explained in section (2). A precise meaning to it can only be given after a regularization procedure is introduced. We

will assume that there is no gauge anomaly in the original theory corresponding to S_0 and we will also take as a prescription that the first term on the right hand side of (24) is to be regularized as in the case without χ . That means that it will contribute at maximum to a trivial (BRST exact) χ independent term that could be absorbed by adding an appropriate counterterm to S_0 . Therefore only the second term on the right hand side of (24) will represent relevant contributions, and consequently we will consider the regularized version of (24) to be just proportional to the new ghosts C^a :

$$(\Delta S)_{Reg.} = i (J_a)_{Reg.} C^a. \quad (25)$$

The important question at this point is: does (25) represent a true gauge anomaly? In the field antifield quantization we say that a theory has a gauge anomaly when, after calculating a regularized ΔS , one verifies that the one loop order master equation (5) does not admit a local solution in the original space of fields and antifields. In other words, when it is not possible to find a counterterm M_1 whose BRST variation is proportional to $(\Delta S)_{Reg.}$. From the cohomological point of view[6], an anomaly corresponds to $(\Delta S)_{Reg.}$ been BRST closed but not BRST exact. However, from the form of the BRST transformations of χ^a and C^a

$$\begin{aligned} \delta_{BRST} \chi^a &= -C^a \\ \delta_{BRST} C^a &= 0 \end{aligned} \quad (26)$$

one realizes that these two fields are absent from the cohomology [6] (they constitute what is called a BRST doublet [12]). That means, eq. (25) represents just a BRST exact term for which one can always find a local M_1 term that solves the master equation (5).

The explicit form of such a counterterm will depend on (25) but it can be put in the form:

$$\hbar M_1 = +\hbar \chi^a (J_a)_{Reg.} |_{\chi=0} + O[\chi^2]. \quad (27)$$

where $O[\chi^2]$ means terms of order two or more in the χ^a fields.

Defining the generating functional as:

$$Z_{\overline{\Psi}}[J_A] = \int [d\varphi^I] \exp \frac{i}{\hbar} (S_{\Sigma} + \hbar M_1[\phi^A, \chi^a] + J_A \phi^A) \quad (28)$$

were $S_{\Sigma} = S[\varphi^I, \varphi_I^* = \frac{\partial \overline{\Psi}}{\partial \varphi^I}]$ and with gauge fixing fermions constrained to the form:

$$\overline{\Psi}[\varphi^I] = \bar{C}_a \chi^a + \Psi[\phi^A] \quad (29)$$

the symmetry (16) is always fixed in the trivial gauge $\chi^a = 0$ and the original theory is recovered.

This answers our question about the existence of gauge anomalies. The theory remains (gauge) anomaly free. This is just what could be expected if $S_0[\phi]$ and

$S_1[\phi, \chi]$ represent the same theory at quantum level. However, the extra contribution M_1 to the quantum action has a non trivial role. As the master equation is satisfied, the change in the functional (28) under small changes $\delta\bar{\Psi}$ in the gauge fixing fermion, for the present case where the classical action is linear in the antifields and M_1 does not involve antifields, is[9]:

$$\begin{aligned} Z_{\bar{\Psi}'}[J_A] - Z_{\bar{\Psi}}[J_A] &= \\ &= \int [d\varphi^I] \exp\left\{\frac{i}{\hbar} \left(S_{\Sigma} + \hbar M_1[\phi^A, \chi^a] + J_A \phi^A\right)\right\} \frac{i}{\hbar} J^A \frac{\partial_l S}{\partial \phi^{*A}} \frac{i}{\hbar} \delta\bar{\Psi} \end{aligned} \quad (30)$$

We will choose the particular variation:

$$\delta\bar{\Psi} = \bar{\Psi}' - \bar{\Psi} = \bar{C}^a \epsilon^a \quad (31)$$

where $\epsilon^a(x)$ are small arbitrary quantities. Furthermore, we will assume that the gauge fixing fermion does not depend on the fields ψ^r (this is what happens, for example, in ref[19], where the gauge fixing part of the action is implicitly assumed not to depend on the fermionic matter fields):

$$\bar{\Psi}[\varphi^I] = \bar{C}_a \chi^a + \Psi[A^m, c_\alpha, \dots] \quad (32)$$

where the dots refer to possible trivial pairs but not to ψ^r .

From this condition and (23) we find

$$\frac{i}{\hbar} \bar{C}^a \exp\frac{i}{\hbar} (S_{\Sigma} + \hbar M_1 + J^A \phi_A) = \frac{\partial}{\partial C^a} \exp\frac{i}{\hbar} (S_{\Sigma} + \hbar M_1 + J^A \phi_A) \quad (33)$$

Partially integrating in C^a and using again (23) we get

$$Z_{\bar{\Psi}'}[J_A] - Z_{\bar{\Psi}}[J_A] = \frac{i}{\hbar} \int [d\varphi^I] \epsilon^a J_r \bar{R}_a^r \exp\frac{i}{\hbar} (S_{\Sigma} + \hbar M_1 + J^A \phi_A) \quad (34)$$

On the other hand, we can explicitly calculate $Z_{\bar{\Psi}'}$ and then, changing variables to $\chi'^a = \chi^a + \epsilon^a$ (which has Jacobian one) and Taylor expanding in this variable get:

$$Z_{\bar{\Psi}'}[J_A] - Z_{\bar{\Psi}}[J_A] = -\frac{i}{\hbar} \int [d\varphi^I] \epsilon^a \frac{\partial}{\partial \chi^a} (S_1 + \hbar M_1) \exp\frac{i}{\hbar} (S_{\Sigma} + M_1 + J^A \phi_A) \quad (35)$$

Note that, as in (28), integration over the $\bar{\pi}^a$ fields will give delta functionals on χ^a that will remove any possible new interactions involving these extra fields.

From (34) and (35) we get:

$$\int [d\varphi^I] \epsilon^a \left\{ \frac{\partial}{\partial \chi^a} (S_1 + \hbar M_1) + J^r \bar{R}_a^r \right\} \exp\frac{i}{\hbar} (S_{\Sigma} + \hbar M_1 + J^A \phi_A) = 0 \quad (36)$$

Now using eqs. (21) and (27) and considering that (36) is valid for arbitrary $\epsilon^a(x)$ we find, after integrating over $\overline{\pi}^a, \overline{C}^a, C^a$, and χ^a :

$$\int [d\phi^A] \left(\partial_\mu J_a^\mu(x) - \hbar (J_a)_{Reg.}|_{\chi=0}(x) - J^r \overline{R}_a^r(x) \right) \exp \frac{i}{\hbar} (S_{BV} + J^A \phi_A) = 0 \quad (37)$$

where

$$S_{BV} = S_{BV}[\phi^A, \phi_A^* = \frac{\partial \Psi}{\delta \phi^A}] = S_0[\phi^i] + \frac{\partial \Psi}{\partial A^m} R_\alpha^m c^\alpha - \frac{\partial \Psi}{\partial c^\alpha} T_{\beta\gamma}^\alpha c^\gamma c^\beta \quad (38)$$

is just the BV action for the original theory, assuming condition (32) to hold. So, we obtain the expectation value (in the original space of fields, with no more χ fields) of the divergence of the Noether's current. Expanding (37) in the sources J_A we get the whole set of Greens functions involving the insertion of the operator $\partial_\mu J_a^\mu$ [26]. This corresponds to the results derived in [19], of course outside of the BV framework. Therefore, the generating functional (28) contains more information than the original BV generating functional of eq. (1), that only contains information about Greens functions involving the original vertices of the theory.

4 Axial Model

Let us begin by considering the model described by the classical action[27]:

$$S_0 = \int d^2x \left(i\overline{\psi}\gamma^\mu \partial_\mu \psi - g_0 \overline{\psi}\gamma^\mu \gamma_5 \psi \partial_\mu \phi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 \right) \quad (39)$$

As action (39) presents no gauge invariance, following standard BV quantization we do not need ghosts at all. The quantum action, to be used in the generating functional would therefore be just (39). This way we would get no information about global anomalies in the model. Let us, however, investigate the two sets of internal rigid symmetries of (39)

$$\begin{aligned} \psi &\rightarrow \psi' = \exp(i\epsilon^1) \psi \\ \overline{\psi} &\rightarrow \overline{\psi}' = \overline{\psi} \exp(-i\epsilon^1) \end{aligned} \quad (40)$$

and

$$\begin{aligned} \psi &\rightarrow \psi' = \exp(i\epsilon^2 \gamma_5) \psi \\ \overline{\psi} &\rightarrow \overline{\psi}' = \overline{\psi} \exp(i\epsilon^2 \gamma_5) \end{aligned} \quad (41)$$

with ϵ^1 and ϵ^2 constants. Of course, we could consider the whole set of global symmetries of (39), but as we are interested in investigating possible anomalies associated to transformations (40) and (41), let us consider only this set. Let us

follow the proposal of section (3) and, in order to investigate the quantum behavior of the Noether currents associated to these global transformations, introduce new fields χ^1 and χ^2 in order to gauge (40) and (41) respectively along the lines described in the previous section. This results in adding to the action (39) the term:

$$- e \int d^2x \left(\bar{\psi} \gamma^\mu \partial_\mu \chi^1 \psi + \bar{\psi} \gamma^\mu \gamma_5 \partial_\mu \chi^2 \psi \right). \quad (42)$$

As the sum of eq. (39) and the term above is invariant now under (40,41) for local ϵ^1 and ϵ^2 once χ^1 and χ^2 transform as $\delta\chi^1 = -\epsilon^1$ and $\delta\chi^2 = -\epsilon^2$, we consider these symmetries in the usual way in the field-antifield formalism. So we include the ghosts C_1 and C_2 corresponding to ϵ^1 and ϵ^2 , and to the previous extended action we add the gauge fixing action

$$\begin{aligned} S_{gf} = & \int d^2x \left[ie \left(\psi^* \psi C_1 - \bar{\psi} \psi^* C_1 + \psi^* \gamma_5 \psi C_2 + \bar{\psi} \gamma_5 \psi^* C_2 \right) \right. \\ & \left. - \chi^{2*} C_2 - \chi^{1*} C_1 + \pi^2 \bar{C}_2^* + \pi^1 \bar{C}_1^* \right]. \end{aligned} \quad (43)$$

Following the ideas introduced in the previous section, we can investigate the anomalies associated to the gauge symmetries so introduced. As the quantum master equation is not well defined, we need to adopt some regularization procedure. A rich regularization scheme (at first order in \hbar) can be given by the Pauli Villars (PV) procedure [7, 9, 28]. In order to properly implement this kind of regularization, we need to introduce PV fields with convenient definitions for the path integral in such a way that the whole measure for the PV fields and the the original ones is BRST invariant. The PV action is also constructed in such a way that the only source of BRST non invariance comes from the PV mass term. If we choose a mass term for the fermionic PV fields that has the usual form as the mass term for Dirac fermions, we can show that after integrating out the PV fields and taking the infinite limit of the regulating mass, we get,

$$(\Delta S)_{Reg.} = -\frac{e}{\pi} \int d^2x C_2 \left(g_0 \square \phi + e \square \chi^2 \right). \quad (44)$$

This result corresponds to the usual Fujikawa regularization where the vectorial transformation is considered as a preferred symmetry. With this in consideration, we see that the master equation at one loop level (5) can be satisfied for this $(\Delta S)_{Reg.}$ choosing the local counterterm

$$M_1 = \frac{ie}{\pi} \int d^2x \left(g_0 \chi^2 \square \phi + \frac{e}{2} \chi^2 \square \chi^2 \right). \quad (45)$$

This shows that the theory has no gauge anomalies, what should be expected from the original theory described by (39). The presence of the counterterm $\hbar M_1$ in the quantum action will however enlarge the content of the generating functional, in the sense that it will also allow the calculation of the quantum expectation values of divergencies of the Noether currents. Writing out equation (37) for $\chi = \chi^2$ and then for $\chi = \chi^1$ and taking the zero order term in the sources, we find respectively:

$$\begin{aligned}
\langle \partial_\mu (\bar{\psi} \gamma^\mu \gamma_5 \psi) \rangle_\Psi &= -i \frac{g_0}{\pi} \langle \square \phi \rangle_\Psi \\
\langle \partial_\mu (\bar{\psi} \gamma^\mu \psi) \rangle_\Psi &= 0
\end{aligned} \tag{46}$$

So the results of ref. [27] are reproduced in the field-antifield formalism, once we gauge the axial symmetry in a minimal way.

As a final comment, we observe that we could have chosen other mass terms. For instance, we could have introduced non transforming PV fields, besides the usual PV fields[9] and easily construct a PV mass term that would be non invariant under the chiral and the vector symmetries. Under these scheme, both symmetries would be non preferential and, as a result, the anomaly would appear along C_2 as well as along C_1 . Because of the cohomological triviality, the master equation would again be satisfied for some M'_1 and following the same procedure used in this section, we would generate an anomalous divergence for the vectorial current as well. These results can also appear in the calculation of anomalous divergencies by following the procedure of Fujikawa.

5 Adler Bell Jackiw Anomaly

Let us consider fermions coupled to non-Abelian gauge fields, described by the action:

$$S_0 = \int d^4x \left[-\frac{1}{4} \text{Tr}(F^{\mu\nu} F_{\mu\nu}) + i \bar{\psi} \gamma^\mu (\partial_\mu - ig A_\mu) \psi \right] \tag{47}$$

with $A_\mu = A_\mu^\alpha T^\alpha$ and $[T^\alpha, T^\beta] = if_{\alpha\beta\gamma} T^\gamma$. This action is invariant under the local infinitesimal transformations:

$$\begin{aligned}
\delta\psi &= ig\omega^\alpha(x) T^\alpha \psi \\
\delta\bar{\psi} &= -ig\bar{\psi}\omega^\alpha(x) T^\alpha \\
\delta A_\mu &= D_\mu \omega
\end{aligned} \tag{48}$$

If we were to follow the steps of the standard BV quantization, we should include just one (non Abelian) ghost, say $c = c^\alpha T^\alpha$, associated to (48). However, we want to investigate the behavior at the quantum level of the global Abelian axial symmetry of (47):

$$\begin{aligned}
\psi' &= \psi_\beta = e^{ig\beta} \psi \\
\bar{\psi}' &= \bar{\psi}_\beta = \bar{\psi} e^{-ig\beta} \\
A'_\mu &= A_\mu
\end{aligned} \tag{49}$$

Thus, following the steps of the previous section, we introduce a bosonic Abelian field χ and an Abelian ghost C associated to the global symmetries (48), writing the total action:

$$\begin{aligned}
S = & \int d^4x \left[-\frac{1}{4} \text{Tr}(F^{\mu\nu} F_{\mu\nu}) + i\bar{\psi}\gamma^\mu(\partial_\mu - igA_\mu + ig\gamma^5\partial_\mu\chi)\psi \right. \\
& + A_\mu^{*\alpha} (D^\mu c)^\alpha - ig\psi^* c\psi + ig\bar{\psi} c\bar{\psi}^* + ig\psi^* \gamma^5 \psi C + ig\bar{\psi} \gamma^5 \bar{\psi}^* C \\
& \left. + \frac{1}{2} c^{*\alpha} c^\beta c^\gamma f_{\alpha\beta\gamma} - \chi^* C + \bar{\pi} \bar{C}^* \right] \quad (50)
\end{aligned}$$

The next step is to calculate ΔS for this enlarged action that includes, besides the original gauge coupling, an additional axial one. This situation, in the field antifield context, was considered in [9], where the results were shown to be equivalent to previous calculations presented in [29] for the fermionic Jacobian, in this case of mixed coupling, using Fujikawa's regularization. Considering, as is the case here, that the axial field is Abelian, their result simplifies to:

$$\begin{aligned}
\Delta S_{Reg.} = & -\frac{g^3}{16\pi^2} \int d^4x C \epsilon^{\mu\nu\rho\sigma} \text{Tr} F_{\mu\nu} F_{\rho\sigma} - \Lambda^2 \frac{g^2}{2\pi^2} \int d^4x C \square \chi \\
& + \frac{g^2}{12\pi^2} \int d^4x C \left(\frac{1}{6} (\square^2 \chi + 4g^2 (\partial_\mu \chi \square \chi \partial^\mu \chi + 2\partial_\mu \partial_\nu \chi \partial^\mu \chi \partial^\nu \chi)) \right) \quad (51)
\end{aligned}$$

where Λ^2 is a regulating parameter. The M_1 term that solves the master equation in this case is:

$$\begin{aligned}
M_1 = & +\frac{ig^3}{16\pi^2} \int d^4x \chi \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \frac{ig^2}{4\pi^2} \int d^4x (\Lambda^2 \chi \square \chi - \frac{1}{6} \chi \square^2 \chi) \\
& - i\frac{g^4}{12\pi^2} \int d^4x (\chi \partial_\mu \chi \square \chi \partial^\mu \chi + 2\chi \partial_\mu \partial_\nu \chi \partial^\mu \chi \partial^\nu \chi) \quad (52)
\end{aligned}$$

The Greens functions with the insertion of $\partial^\mu J_{\mu 5}$ can then be calculated from eq. (37), considering a source term like :

$$J^A \phi_A = \bar{\psi} \eta + \bar{\eta} \psi + J^\mu A_\mu \quad (53)$$

and then expanding in the sources.

One example is:

$$\begin{aligned}
\langle \partial^\mu J_{\mu 5}(x) \psi(y) \bar{\psi}(z) \rangle_\Psi = & \langle \hbar \frac{ig^3}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(x) \psi(y) \bar{\psi}(z) \rangle_\Psi \\
& + ig\delta(x-y) \langle \gamma_5 \psi(y) \bar{\psi}(z) \rangle_\Psi + ig\delta(x-z) \langle \psi(y) \bar{\psi}(z) \gamma_5 \rangle_\Psi \quad (54)
\end{aligned}$$

as in references [17, 19].

6 Conclusions

Global Anomalies play a very important role in the description of processes like the π_o decay[18]. In this way it is interesting to build up a generating functional that describes this kind of behavior. In the standard formulation, the Noether currents associated to global transformations are in general not present in the Lagrangian, in contrast to the local currents, which are coupled to the gauge fields. That is why the standard BV, in general, does not allow computations of Greens functions involving global currents. We have shown in this article that for global Abelian symmetries, the generating functional in the BV quantization procedure can be built up with extra fields, together with associated extra gauge degrees of freedom, in such a way that global anomalies naturally arise from the generating functional. It is important to mention that our approach has some similarities with the field space enlargement used in ref. [30]. There, the gauge symmetry group of some theory is trivially extended and then an appropriate gauge fixing of the extra symmetries leads to the BV action with an interesting interpretation for the antifields.

Our results were obtained for the case of global Abelian symmetries. Also only irreducible theories with closed algebra were considered. More general situations are under study and the results will be reported elsewhere.

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